**7.3 and 7.4 – Creating Rules to Define Sequences**

1. Alternating Patterns of Adding and Subtracting

Patterns can exist where there are two different common differences between alternating terms. For example, add 5, subtract 2, add 5, subtract 2, etc.

**Example One**

Identify the next three terms in the following sequences.

a) 5, 11, 9, 15, 13, 19, 17, … b) 22, 9, 18, 5, 14, 1, 10, …

1. Examining Patterns in the First and Second Differences

Sometimes patterns will exist in the first and second differences, which can be used to determine the next term(s) in the sequence.

**Example Two**

Examine the first differences of the following sequence and identify if a pattern exists. If a pattern does exist, identify the next three terms of the sequence.

1, 8, 16, 26, 39, 56, 78, …

1. Patterns Involving Fractions

When the pattern involves fractions, it is often useful to look for patterns in the numerator and patterns in the denominator separately.

**Example Four**

Determine the general term of the following sequence and use it to determine the 20th term of this sequence.

$$\frac{3}{4}, \frac{5}{9}, \frac{7}{16}, \frac{9}{25}, \frac{11}{36}, \frac{13}{49}, \frac{15}{64}, …$$

Recursive Sequences with Patterns Involving Multiplication and Addition

One of the most difficulty types of patterns to recognize are ones in which there exists a pattern of multiplication ANDaddition/subtraction. For example, the sequence:

7, 27, 107, 427, 1707, …

contains a pattern in which the first term is multiplied by 4 then 1 is subtracted from this product to get the next term.

The recursive sequence for this pattern would be: t1 = 7, tn = 4tn-1 – 1

When patterns involve both multiplication and addition/subtraction, the ratio of any term in the sequence to the previous term will come closer and closer to some number. This number is the number being multiplied each time.

For example, looking back at the example, 7, 27, 107, 427, 1707

**Example Three**

Determine the recursive sequence of the following patterns and use it to determine the next three terms in the sequence.

a) 5, 14, 41, 122, 365, … b) 12, 41, 157, 621, 2477, …

The Fibonacci Sequence

The Fibonacci Sequence is one of the most famous examples of a recursive sequence. This sequence was first discovered by Leonardo Pisano (whose nickname was Fibonacci). The sequence is based on the mating patterns of rabbits. See p. 441 of the textbook.

The Fibonacci sequence follows the pattern: 1, 1, 2, 3, 5, 8, 13, 21, 34, …

**Example One**

Determine a recursive sequence that would model this situation.